

BABYLONIAN Tonal System

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Abstract

Proposal for the BABYLONIAN Tonal System, based on consecutive heptachords.¹

Keywords. Ancient MESOPOTAMIAN music, Old BABYLONIAN music, MESOPOTAMIA, BABYLON, UR, SUMER, AKKADIAN, 1800 BCE to 500 BCE, Sexagesimal arithmetic and numbers, Musical cuneiform tablets, Seven diatonic Heptachords, Tuning system.

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‡Revised version.

¹Original research, based on [1] and an idea by composer CARL NIELSEN (1865-1931).

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Part I

Heptachords and Scales

1 Introduction

1.1 Heptachords

Table 1: Heptachords

	Heptachord	Pattern	Scales	Symmetric Center
1	<i>išartum</i>	$[1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}]$	2	G
2	<i>embūbum</i>	$[\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 1]$	2	A
3	<i>kitmum</i>	$[1, \frac{1}{2}, 1, 1, \frac{1}{2}, 1]$	2	D
4	<i>pītum</i>	$[\frac{1}{2}, 1, 1, \frac{1}{2}, 1, 1]$	2	E
5	<i>qablītum</i>	$[1, 1, \frac{1}{2}, 1, 1, \frac{1}{2}]$	2	C
6	<i>nīš tuḥrim</i>	$[1, 1, \frac{1}{2}, 1, 1, 1]$	1	F
7	<i>nīd qablim</i>	$[1, 1, 1, \frac{1}{2}, 1, 1]$	1	B

2

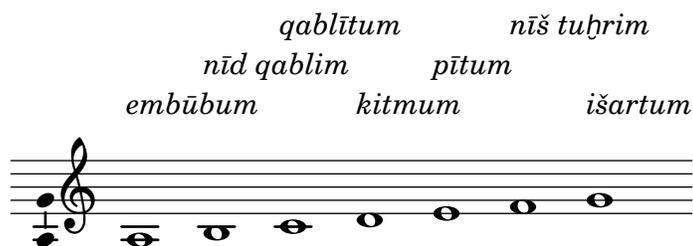


Figure 1: Symmetric Center

² Rising Scale.

Table 2: Heptachords II

	Heptachord	Keys per Scale	Key Signature Δ	Sum Σ
1	<i>išartum</i>	6	2	12
2	<i>embūbum</i>	6	2	12
3	<i>kitmum</i>	6	2	12
4	<i>pītum</i>	6	2	12
5	<i>qablātum</i>	6	2	12
6	<i>nīš tuḫrim</i>	12	5	60
7	<i>nād qablim</i>	12	5	60

1: 𐎗

2: 𐎗

5: 𐎗

6: 𐎗

12: 𐎗

60: 𐎗 ANU

Example. Consecutive Heptachords.^{3,4}

The image shows a musical staff with a treble clef. It displays three consecutive heptachords, each consisting of seven notes. The first heptachord is labeled "D-*kitmum* (1 ♭)" and starts on D4. The second is labeled "C-*kitmum* (3 ♭)" and starts on C4. The third is labeled "B♭-*kitmum* (5 ♭)" and starts on B♭4. The notes are connected by semitone intervals, as indicated by brackets labeled "semitone" between each pair of adjacent notes. The notes are: D4, E4, F4, G4, A4, B4, C5 for the first; C4, D4, E4, F4, G4, A4, B4 for the second; and B♭4, C5, D5, E5, F5, G5, A5 for the third. The notes are written as half notes.

Figure 2: Consecutive *kitmum* Heptachords

³ From the main theme of CARL NIELSEN *Symphony No. 5* CNW 29 (1922).

⁴ Flatward shift for higher pitches.

1.2 Scale Range

Table 3: Scale Range

	Heptachord	Center	Semitones	Octaves	i , Cent	Cent per semitone
1	<i>išartum</i>	G	60	5	6000	100
2	<i>embūbum</i>	A	60	5	6000	100
3	<i>kitmum</i>	D	60	5	6000	100
4	<i>pītum</i>	E	60	5	6000	100
5	<i>qablītum</i>	C	60	5	6000	100
6	<i>nāš tuhrim</i>	F	120	10	12000	100
7	<i>nād qablim</i>	B	120	10	12000	100

Definition. Cent per octave: 1200.

Definition. Cent per semitone: 100, Equal temperament (12-TET⁵ or 12-EDO⁶).

Definition. Scale Range: $-3000 \dots 0$ (Center) $\dots +3000$ Cent.⁷

Theorem. *The 7 Heptachords of the Babylonian Tonal System are based on an Equal Temperament system.*

⁵ Twelve-tone equal temperament.

⁶ Equal division of the octave.

⁷ Semitone: $i \leftarrow i + 100$, $i \leftarrow i - 100$. Whole tone: $i \leftarrow i + 200$, $i \leftarrow i - 200$.

1.3 Scale Cycle

Table 4: Scale Cycle

	Heptachord	Center	+3000 Cent	-3000 Cent
1	<i>išartum</i>	G	D \flat	C \sharp
2	<i>embūbum</i>	A	E \flat	D \sharp
3	<i>kitnum</i>	D	A \flat	G \sharp
4	<i>pītum</i>	E	B \flat	A \sharp
5	<i>qablītum</i>	C	G \flat	F \sharp

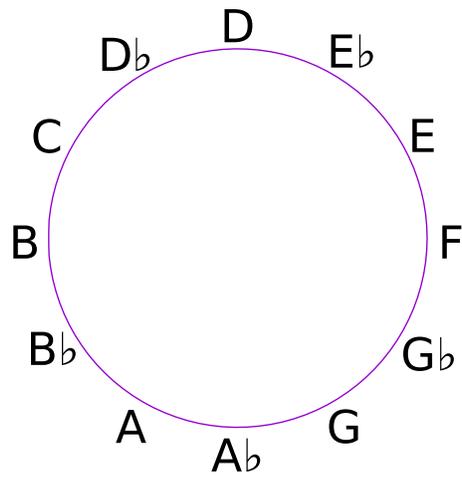


Figure 3: Tone Circle

1.4 Tuning

Conjecture. *Equal Temperament Tuning Procedure.*

Table 5: Tuning

	Heptachord	Center	i , Cent	f , Hz
1	<i>išartum</i>	G4	+500	
2	<i>embūbum</i>	A3	-500	220
3	<i>kitmum</i>	D4	0	
4	<i>pītum</i>	E4	+200	
5	<i>qablītum</i>	C4	-200	
6	<i>nāš tuḫrim</i>	F4	+300	
7	<i>nād qablim</i>	B3	-300	

8

Definition. Tuning Range: -500 ... 0 ... +500 Cent.

⁸ Center in Scientific Pitch Notation.

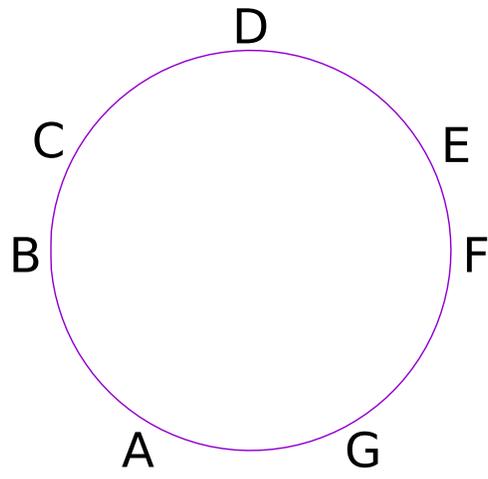


Figure 4: Tuning

2 *išartum*

$[1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}]$

2.1 Scale 1

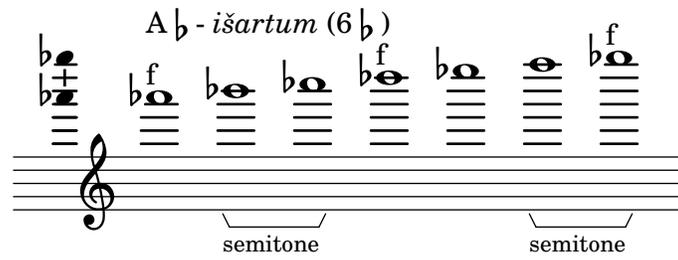


Figure 5: Scale I

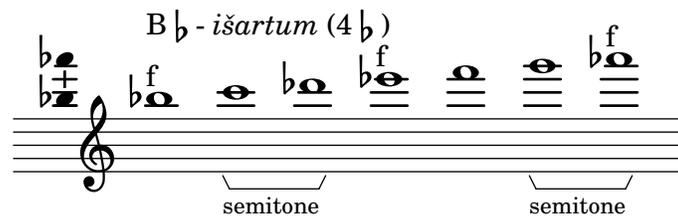


Figure 6: Scale II

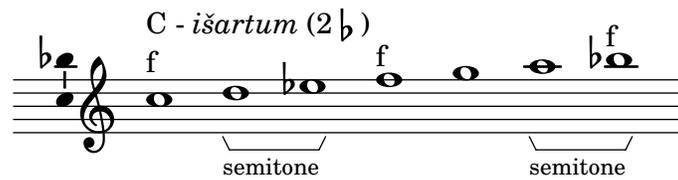


Figure 7: Scale III

D - *išartum* (0 ♭)

Figure 8: Scale IV

E - *išartum* (2 ♯)

Figure 9: Scale V

F ♯ - *išartum* (4 ♯)

Figure 10: Scale VI

G ♯ - *išartum* (6 ♯)

Figure 11: Scale VII

2.2 Scale 2

$D \flat$ - *išartum* (7 \flat)

semitone semitone

Figure 12: Scale I

$E \flat$ - *išartum* (5 \flat)

semitone semitone

Figure 13: Scale II

F - *išartum* (3 \flat)

semitone semitone

Figure 14: Scale III

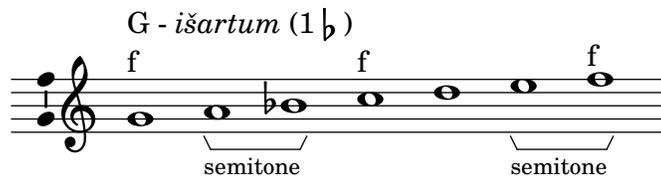


Figure 15: Scale IV

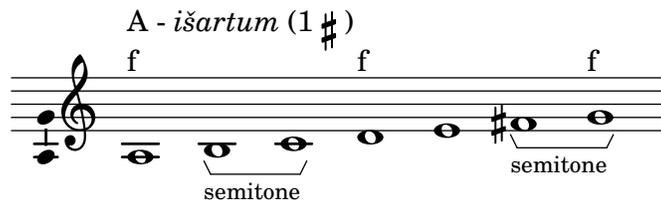


Figure 16: Scale V

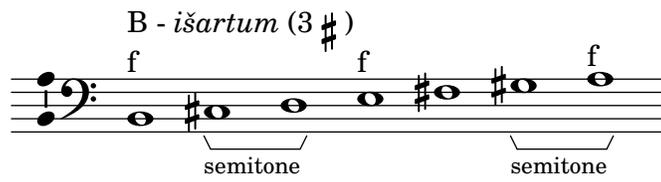


Figure 17: Scale VI

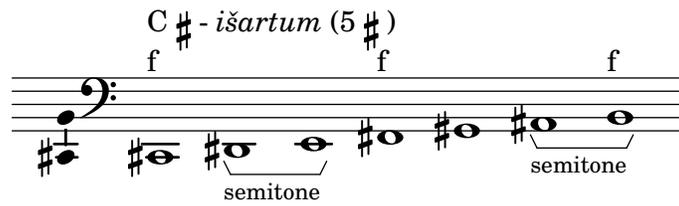


Figure 18: Scale VII

3 *embūbum*

$[\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 1]$

3.1 Scale 1

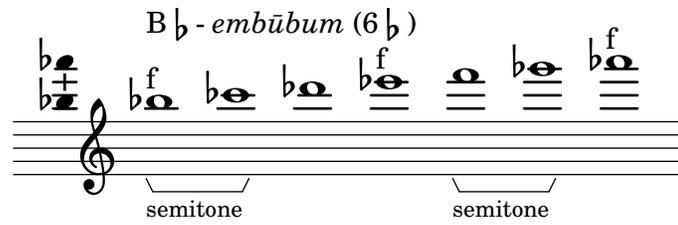


Figure 19: Scale I

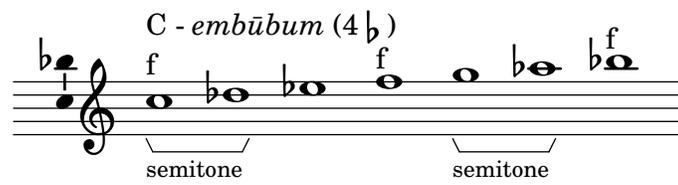


Figure 20: Scale II

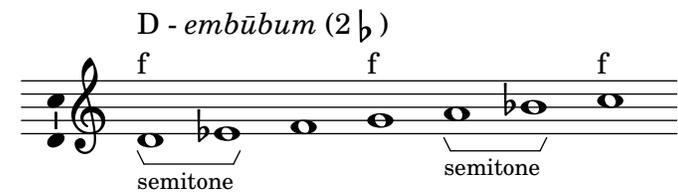


Figure 21: Scale III

E - *embūbum* (0 ♭)

f f f

semitone

semitone

Figure 22: Scale IV

F # - *embūbum* (2 #)

f f f

semitone

semitone

Figure 23: Scale V

G # - *embūbum* (4 #)

f f f

semitone

semitone

Figure 24: Scale VI

A # - *embūbum* (6 #)

f f f

semitone

semitone

Figure 25: Scale VII

3.2 Scale 2

E \flat - *embūbum* (7 \flat)

semitone semitone

Figure 26: Scale I

F - *embūbum* (5 \flat)

semitone semitone

Figure 27: Scale II

G - *embūbum* (3 \flat)

semitone semitone

Figure 28: Scale III

A - *embūbum* (1 ♭)

Figure 29: Scale IV

B - *embūbum* (1 ♯)

Figure 30: Scale V

C - *embūbum* (3 ♯)

Figure 31: Scale VI

D ♯ - *embūbum* (5 ♯)

Figure 32: Scale VII

4 *kitmum*

$[1, \frac{1}{2}, 1, 1, \frac{1}{2}, 1]$

4.1 Scale 1

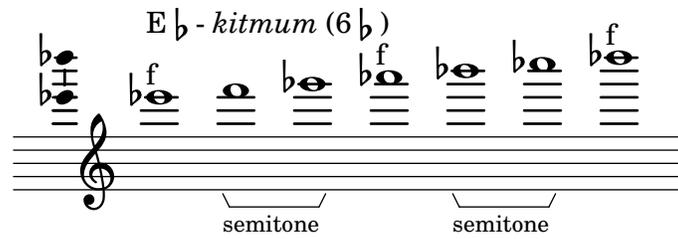


Figure 33: Scale I

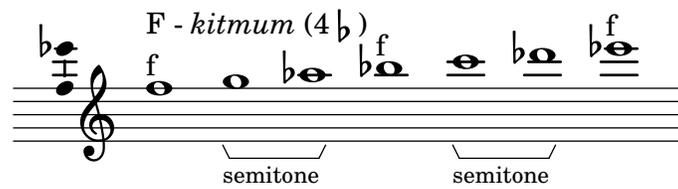


Figure 34: Scale II

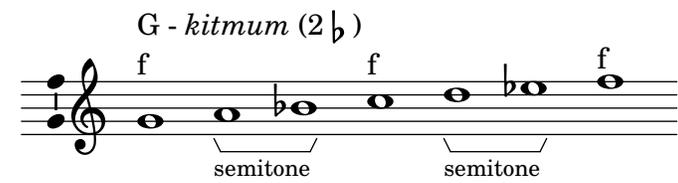


Figure 35: Scale III

A - *kitmum* (0 ♭)

Figure 36: Scale IV

B - *kitmum* (2 ♯)

Figure 37: Scale V

C ♯ - *kitmum* (4 ♯)

Figure 38: Scale VI

D ♯ - *kitmum* (6 ♯)

Figure 39: Scale VII

4.2 Scale 2

A \flat - *kitmum* (7 \flat)

semitone semitone

Figure 40: Scale I

B \flat - *kitmum* (5 \flat)

semitone semitone

Figure 41: Scale II

C - *kitmum* (3 \flat)

semitone semitone

Figure 42: Scale III

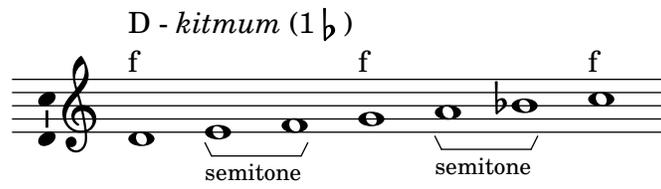


Figure 43: Scale IV

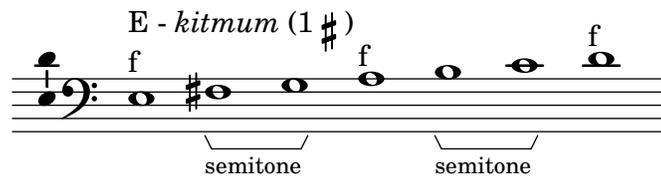


Figure 44: Scale V

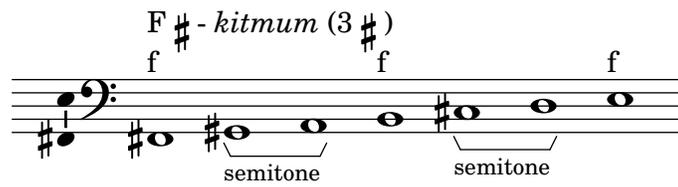


Figure 45: Scale VI

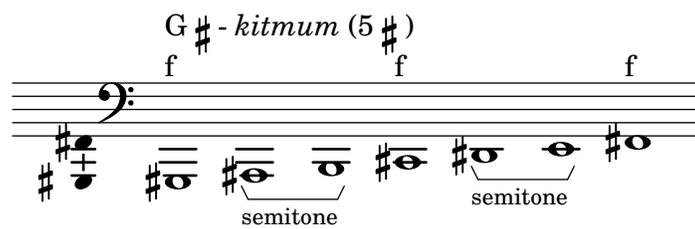


Figure 46: Scale VII

5 *pītum*

$[\frac{1}{2}, 1, 1, \frac{1}{2}, 1, 1]$

5.1 Scale 1

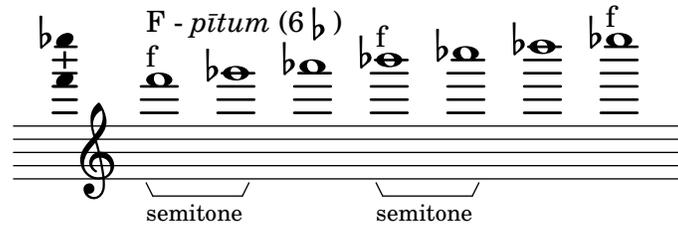


Figure 47: Scale I

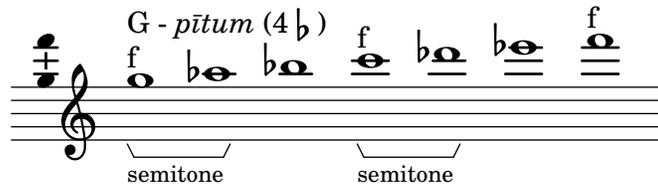


Figure 48: Scale II

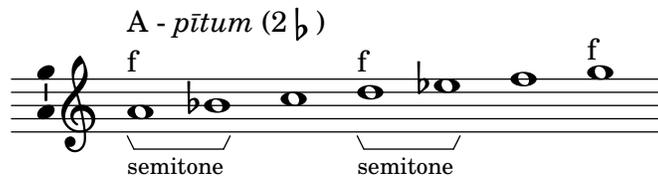


Figure 49: Scale III

B - *pītum* (0 ♭)

f f f

semitone semitone

Figure 50: Scale IV

C # - *pītum* (2 #)

f f f

semitone semitone

Figure 51: Scale V

D # - *pītum* (4 #)

f f f

semitone semitone

Figure 52: Scale VI

E # - *pītum* (6 #)

f f f

semitone semitone

Figure 53: Scale VII

5.2 Scale 2

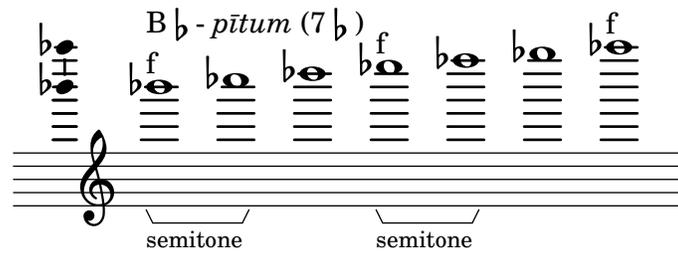


Figure 54: Scale II

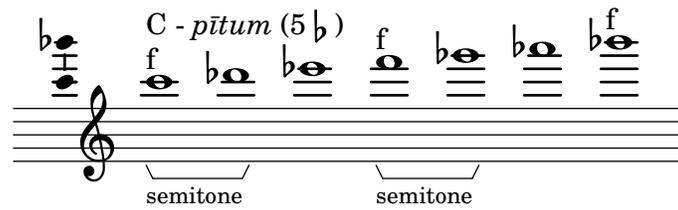


Figure 55: Scale II

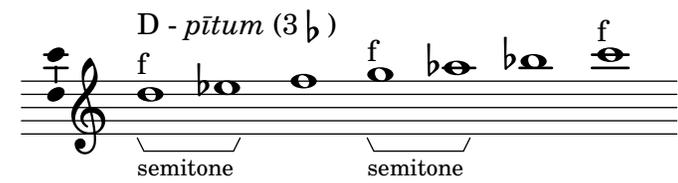


Figure 56: Scale III

E - *pītum* (1 ♭)

Figure 57: Scale IV

F # - *pītum* (1 #)

Figure 58: Scale V

G # - *pītum* (3 #)

Figure 59: Scale VI

A # - *pītum* (5 #)

Figure 60: Scale VII

6 *qablītum*

[1, 1, $\frac{1}{2}$, 1, 1, $\frac{1}{2}$]

6.1 Scale 1

D \flat - *qablītum* (6 \flat)

semitone semitone

Figure 61: Scale I

E \flat - *qablītum* (4 \flat)

semitone semitone

Figure 62: Scale II

F - *qablītum* (2 \flat)

semitone semitone

Figure 63: Scale III

G - *qablītum* (0 ♭)

Figure 64: Scale IV

A - *qablītum* (2 ♯)

Figure 65: Scale V

B - *qablītum* (4 ♯)

Figure 66: Scale VI

C ♯ - *qablītum* (6 ♯)

Figure 67: Scale VII

6.2 Scale 2

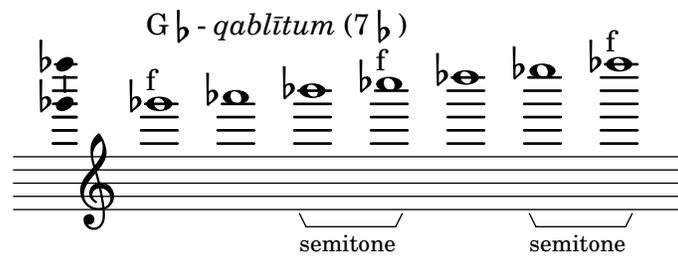


Figure 68: Scale I

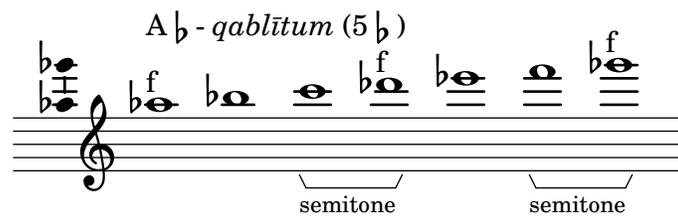


Figure 69: Scale II

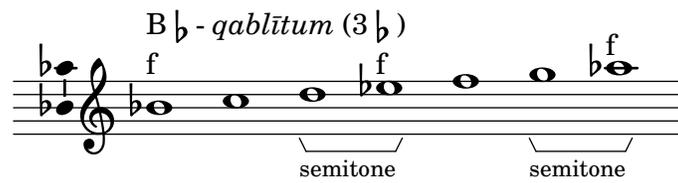


Figure 70: Scale III

C - *qablītum* (1 ♭)

f f f

semitone semitone

Figure 71: Scale IV

D - *qablītum* (1 #)

f f f

semitone semitone

Figure 72: Scale V

E - *qablītum* (3 #)

f f f

semitone semitone

Figure 73: Scale VI

F # - *qablītum* (5 #)

f f f

semitone semitone

Figure 74: Scale VII

7 *nīš tuḥrim*

[1, 1, $\frac{1}{2}$, 1, 1, 1]

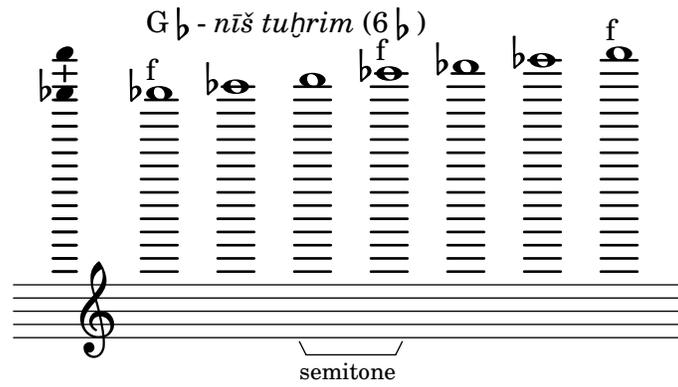


Figure 75: Scale I

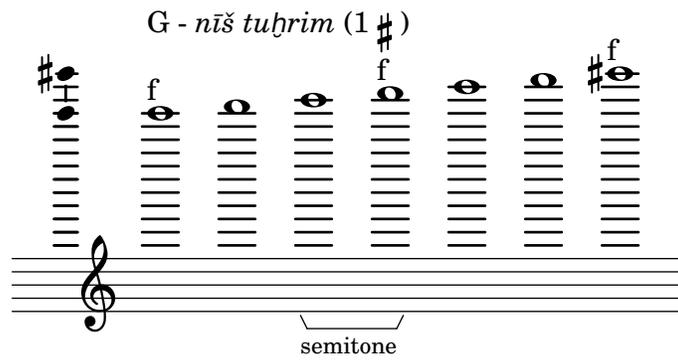


Figure 76: Scale II

A \flat - *nīš tuḥrim* (4 \flat)

semitone

Figure 77: Scale III

A - *nīš tuḥrim* (3 \sharp)

semitone

Figure 78: Scale IV

B \flat - *nīš tuḥrim* (2 \flat)

semitone

Figure 79: Scale V

B - *nīš tuḥrim* (5 \sharp)

semitone

Figure 80: Scale VI

C - *nīš tuḥrim* (0 ♭)

Figure 81: Scale VII

D ♭ - *nīš tuḥrim* (5 ♭)

Figure 82: Scale VIII

D - *nīš tuḥrim* (2 ♯)

Figure 83: Scale IX

E ♭ - *nīš tuḥrim* (3 ♭)

Figure 84: Scale X

E - *nīš tuḥrim* (4 #)

f f f

semitone

Figure 85: Scale XI

F - *nīš tuḥrim* (1 ♭)

f f f

semitone

Figure 86: Scale XII

F # - *nīš tuḥrim* (6 #)

f # f # f

semitone

Figure 87: Scale XIII

8 *nīd qablim*

[1, 1, 1, $\frac{1}{2}$, 1, 1]

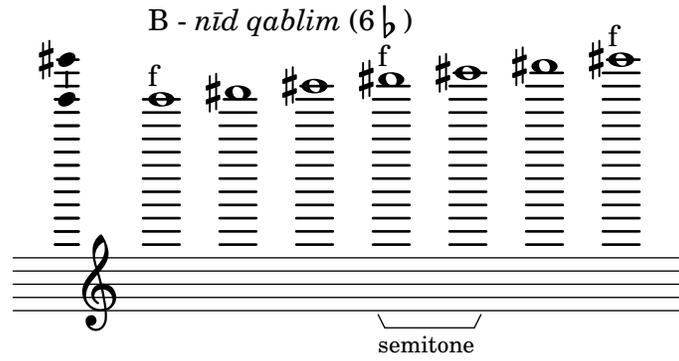


Figure 88: Scale I

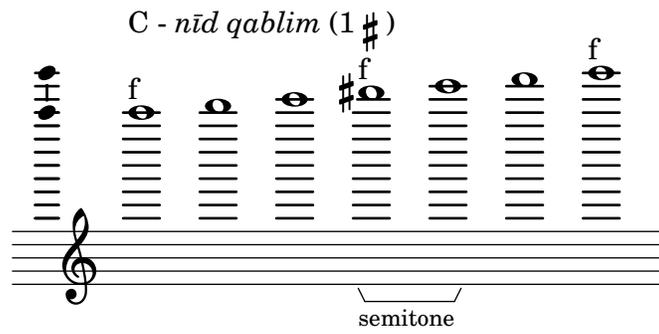


Figure 89: Scale II

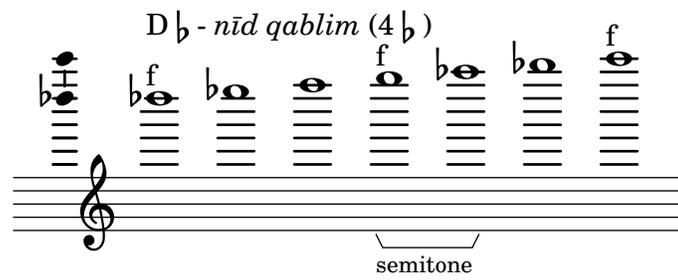


Figure 90: Scale III

D - *nīd qablim* (3 #)

Figure 91: Scale IV

E \flat - *nīd qablim* (2 \flat)

Figure 92: Scale V

E - *nīd qablim* (5 #)

Figure 93: Scale VI

F - *nīd qablim* (0 \flat)

Figure 94: Scale VII

References

- [1] Leon Crickmore. New Light on the Babylonian Tonal System. In Richard Dumbrill and Irving Finkel, editors, *ICONEA 2008: Proceedings of the International Conference of Near Eastern Archaeomusicology*, volume 24, pages 11–22. Lulu, 2008. Held at the British Museum, December 4-6. 1

A Appendix

A.1 ~~Octatonic~~ Scale

9

A.1.1 Scale 1

$[1, \frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{2}]$ ¹⁰

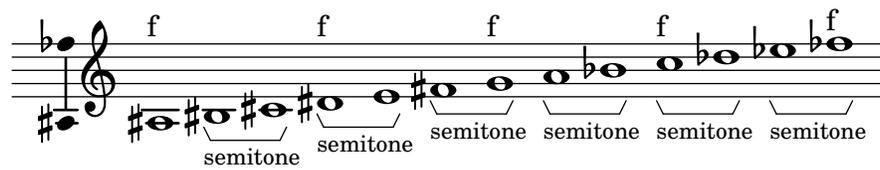


Figure 101: Scale 1, Center G

A.1.2 Scale 2

$[\frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{2}, 1]$

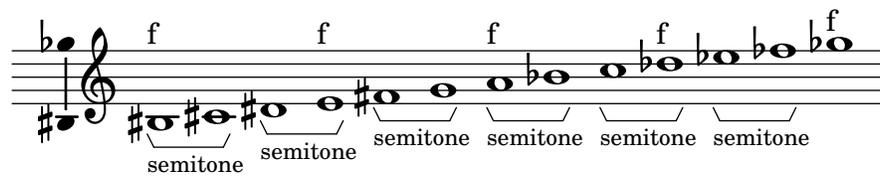


Figure 102: Scale 2, Center A

⁹ Derived from CARL NIELSEN *Symphony No. 5* CNW 29.

¹⁰ OLIVIER MESSIAEN's *second mode* of limited transposition.

A.2 Plots

A.2.1 Heptachords

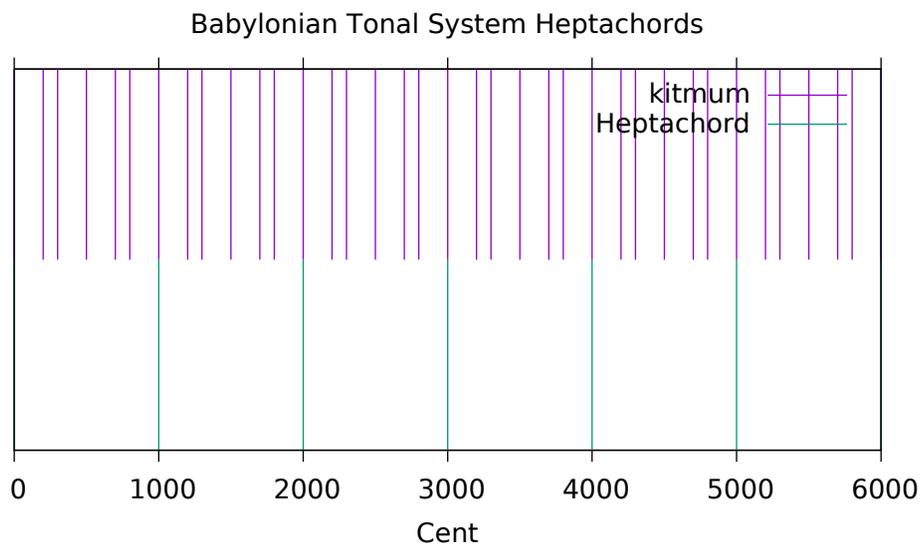


Figure 103: *kitnum* vs. Heptachord

A.2.2 Octaves

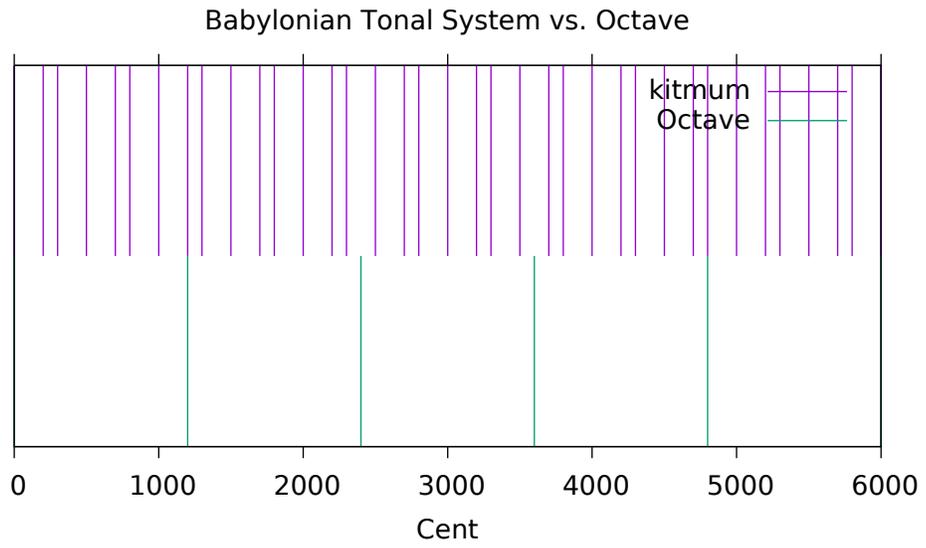


Figure 104: *kitmum* vs. Octave